

Extracting the transversity distributions from single-hadron and dihadron production

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We present a point-by-point determination of the valence transversity distributions from two different types of processes: single-hadron production and dihadron production, both in semi-inclusive deep inelastic scattering and e^+e^- annihilation. The extraction is based on some simple assumptions and does not require any parametrization. The transversity distributions obtained from Collins effect in single-hadron production and from interference effects in dihadron production are found to be compatible with each other.

PACS numbers: 13.88.+e, 13.60.-r, 13.66.Bc, 13.85.Ni

I. INTRODUCTION

The transversity distribution, usually called h_1 , is a leading-twist distribution function that measures the transverse polarization of quarks inside a transversely polarized nucleon (for reviews, see [1, 2]). Introduced already 35 years ago by Ralston and Soper [3], its importance was recognized only recently [4]. It is related to the tensor charge of the nucleon and its difference from the helicity distribution quantifies the relativistic effects in the hadronic structure.

Being chirally odd, transversity decouples from deep inelastic scattering and in order to detect it one has to look either at hadron-hadron scattering or at hadron lepton-pair production. Drell-Yan lepton-pair production with both colliding hadrons transversely polarized involves h_1 only, but no measurements of this process exist yet.

Single-spin asymmetries clearly related to the transversity distribution function have been measured over the past ten years in semi-inclusive deep inelastic scattering (SIDIS) on transversely polarized nucleons, assessing beyond any doubt that transversity is not zero [5–11].

Two processes involving the transversity distributions have been explored so far. The first one is single-hadron production in SIDIS with a transversely polarized target. In this process, transversity couples to a transverse-momentum dependent chirally odd fragmentation function, H_1^\perp , the so-called Collins function [12]. This function is independently probed in hadron pair production in e^+e^- annihilation, where it emerges in a particular azimuthal correlation of the final hadrons.

The second process probing the transversity distributions is dihadron production in SIDIS with a transversely polarized target [13, 14]. In this case, transversity couples to a dihadron fragmentation function, H_1^{\triangleleft} , and the main advantage is that it appears collinearly, without involving any transverse momentum [15]. Again, the fragmentation function H_1^{\triangleleft} can be independently obtained from dihadron pair production in e^+e^- annihilation.

Recent work points to a close relationship between the Collins and the dihadron fragmentation functions [11, 16, 17]. However, in the present paper the two processes will be treated independently as it has been done so far.

Two phenomenological collaborations have extracted the valence u and d transversity distributions by fitting the SIDIS and the e^+e^- annihilation asymmetries. The Torino group [18–20] has used only single-hadron data, while the Pavia group [21, 22] has used only dihadron data. The fits of these groups are based on different hypotheses, but their results are compatible within the present uncertainties.

In this paper we will follow a different approach. Our aim is to extract the transversity point by point in x both from single-hadron and dihadron data. In particular, we will determine from the e^+e^- measurements the analyzing power of transversely polarized single-hadron and dihadron production, and then use this information to get the transversity distributions from the SIDIS data. Since we need the widest set of SIDIS observables to extract $h_1^{u_v}$ and $h_1^{d_v}$ at the same x and Q^2 values, we use the COMPASS data, which are available both for a proton and a deuteron target [8, 10, 11, 23]. As for e^+e^- scattering, we use the Belle data [24–26]. Recently the BaBar Collaboration has also provided data on the Collins

asymmetry [27], but the corresponding data for the dihadron production are not yet available.

The main advantage of our approach is that we can analyze single-hadron and dihadron data in a similar way, keeping all sources of uncertainty under control and easily checking the robustness of the results. We will not need to introduce any parametrization of the data. This considerably simplifies the determination of the transversity, but, on the other hand, prevents us from evolving the distributions.

The use of consistent assumptions for the two sets of processes (single-hadron and dihadron) allows a direct comparison of the transversity distributions obtained in the two cases. As we will see, the valence transversities obtained from the transverse-momentum-dependent factorization of single-hadron production and from the collinear factorization of dihadron production turn out to be mutually compatible. This is also a check of the validity of the two types of factorization.

The plan of the paper is the following. We start, in Sec. II, from dihadron asymmetries, which allow a simpler analysis. In Sec. III we carry out a similar analysis for the Collins asymmetries in single-hadron production and compare the results with those obtained in the dihadron case. Sec. IV contains a general discussion and some concluding remarks.

II. DIHADRON ASYMMETRIES

A. Dihadron asymmetries in SIDIS

The transversity distributions can be probed in dihadron leptonproduction from a transversely polarized target, $\ell N^\uparrow \rightarrow \ell' (h_1 h_2) X$, with the two spinless hadrons (typically pions) in the current jet [13–15]. The idea is to look at an angular correlation between the spin of the fragmenting quark and the relative transverse momentum of the hadron pair. Integrating over the total transverse momentum of the final hadrons, one gets a transverse target spin asymmetry in the azimuthal angle between a well-defined two-hadron plane and the scattering plane. This asymmetry couples the transversity to the dihadron fragmentation function H_1^\lessgtr and does not involve the quark transverse momentum. H_1^\lessgtr , which originates from the interference of different channels in the dihadron formation, describes the fragmentation of a transversely polarized quark into a pair of spinless hadrons.

The kinematics of the process in the $\gamma^* N$ frame involves the following variables: the total momentum of the hadron pair $P_{hh} = P_1 + P_2$ (with invariant mass $M_{hh}^2 = P_{hh}^2$), the relative momentum $R = (P_1 - P_2)/2$, and the light-cone variables $z_{hh} = P_{hh}^-/\kappa^-$ (which is the fraction of the longitudinal momentum κ^- of the fragmenting quark carried by the hadron pair) and $\zeta = 2R^-/P_{hh}^-$ (which describes how the total longitudinal momentum of the pair is split into the two hadrons). \mathbf{R}_T is the transverse component of R with respect to \mathbf{P}_{hh} , and ϕ_R is the azimuthal angle of \mathbf{R}_T in the plane orthogonal to the $\gamma^* N$ axis, measured with respect to the scattering plane. The azimuthal angle of the target spin vector is ϕ_S . Notice that the COMPASS collaboration uses, instead of \mathbf{R}_T , the vector $\mathbf{R}_\perp = (z_2 \mathbf{P}_{1\perp} - z_1 \mathbf{P}_{2\perp})/(z_1 + z_2)$, which is perpendicular to the $\gamma^* N$ axis. The azimuthal angle of this vector, which is the relevant variable for selecting the asymmetry we are interested in, is the same as ϕ_R . A discussion on the different definitions of the dihadron azimuthal angle can be found in Ref. [28].

The term in the cross section containing the transversity is characterized by the angular modulation $\sin(\phi_R + \phi_S)$ and the corresponding measured asymmetry is

$$A^{hh}(x, z_{hh}, M_{hh}^2, Q^2) = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q(x, Q^2) \frac{|\mathbf{R}_T|}{M_{hh}} H_{1q}^\lessgtr(z_{hh}, M_{hh}^2, Q^2)}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x, Q^2) D_{1q}^{hh}(z_{hh}, M_{hh}^2, Q^2)}, \quad (1)$$

where $f_1(x, Q^2)$ is the unpolarized distribution function and $D_{1q}^{hh}(z_{hh}, M_{hh}^2, Q^2)$ is the unpolarized dihadron fragmentation function.

We now incorporate $|\mathbf{R}_T|/M_{hh}$ into H_1^\lessgtr and integrate over z_{hh} and M_{hh}^2 :

$$\tilde{D}_{1q}^{hh}(Q^2) = \int dz_{hh} \int dM_{hh}^2 D_{1q}^{hh}(z_{hh}, M_{hh}^2, Q^2), \quad (2)$$

$$\tilde{H}_{1q}^\lessgtr(Q^2) = \int dz_{hh} \int dM_{hh}^2 \frac{|\mathbf{R}_T|}{M_{hh}} H_{1q}^\lessgtr(z_{hh}, M_{hh}^2, Q^2). \quad (3)$$

The asymmetry can thus be written as

$$A^{hh}(x, Q^2) = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q(x, Q^2) \tilde{H}_{1q}^\lessgtr(Q^2)}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x, Q^2) \tilde{D}_{1q}^{hh}(Q^2)}, \quad (4)$$

Isospin symmetry and charge conjugation suggest the following relations [21, 29]:

$$\tilde{D}_{1u}^{hh} = \tilde{D}_{1d}^{hh} = \tilde{D}_{1\bar{u}}^{hh} = \tilde{D}_{1\bar{d}}^{hh}, \quad \tilde{D}_{1s}^{hh} = \tilde{D}_{1\bar{s}}^{hh}, \quad \tilde{D}_{1c}^{hh} = \tilde{D}_{1\bar{c}}^{hh}, \quad (5)$$

$$\tilde{H}_{1u}^\lessgtr = -\tilde{H}_{1d}^\lessgtr = -\tilde{H}_{1\bar{u}}^\lessgtr = \tilde{H}_{1\bar{d}}^\lessgtr, \quad \tilde{H}_{1s}^\lessgtr = -\tilde{H}_{1\bar{s}}^\lessgtr = \tilde{H}_{1c}^\lessgtr = -\tilde{H}_{1\bar{c}}^\lessgtr = 0. \quad (6)$$

We also set $\tilde{D}_{1s}^{hh} = \lambda \tilde{D}_{1u}^{hh}$, where λ is a numerical factor which is expected to be smaller than unity. In the following we will fix it to 0.5 according to the Monte Carlo simulation of Ref. [21].

Using the relations (5,6) and neglecting the charm distribution function, the asymmetry for the proton target reduces to

$$A_p^{hh}(x, Q^2) = \frac{4xh_1^{uv}(x, Q^2) - xh_1^{dv}(x, Q^2)}{4xf_1^{u+\bar{u}}(x, Q^2) + xf_1^{d+\bar{d}}(x, Q^2) + \lambda xf_1^{s+\bar{s}}(x, Q^2)} \frac{\tilde{H}_{1u}^\lessgtr(Q^2)}{\tilde{D}_{1u}^{hh}(Q^2)} \quad (7)$$

where $f_1^{q+\bar{q}} \equiv f_1^q + f_1^{\bar{q}}$ and $h_1^{qv} = h_1^q - h_1^{\bar{q}}$. For the deuteron target the asymmetry is

$$A_d^{hh}(x, Q^2) = \frac{3xh_1^{uv}(x, Q^2) + 3xh_1^{dv}(x, Q^2)}{5xf_1^{u+\bar{u}}(x, Q^2) + 5xf_1^{d+\bar{d}}(x, Q^2) + 2\lambda xf_1^{s+\bar{s}}(x, Q^2)} \frac{\tilde{H}_{1u}^\lessgtr(Q^2)}{\tilde{D}_{1u}^{hh}(Q^2)}. \quad (8)$$

From Eqs. (7) and (8) one can obtain the following combinations of the valence transversity distributions (for simplicity we omit the dependence on x and Q^2) [21, 22, 30]

$$4xh_1^{uv} - xh_1^{dv} = \frac{1}{\tilde{a}_P^{hh}} \left(4xf_1^{u+\bar{u}} + xf_1^{d+\bar{d}} + \lambda xf_1^{s+\bar{s}} \right) A_p^{hh}, \quad (9)$$

$$xh_1^{uv} + xh_1^{dv} = \frac{1}{3} \frac{1}{\tilde{a}_P^{hh}} \left(5xf_1^{u+\bar{u}} + 5xf_1^{d+\bar{d}} + 2\lambda xf_1^{s+\bar{s}} \right) A_d^{hh}, \quad (10)$$

where the analyzing power \tilde{a}_P^{hh} of the process is defined as

$$\tilde{a}_P^{hh}(Q^2) = \frac{\tilde{H}_{1u}^\lessgtr(Q^2)}{\tilde{D}_{1u}^{hh}(Q^2)}. \quad (11)$$

By combining the proton and the deuteron asymmetries one can extract the transversity distributions for each flavor:

$$xh_1^{uv} = \frac{1}{15} \frac{1}{\tilde{a}_P^{hh}} \left[3 \left(4xf_1^{u+\bar{u}} + xf_1^{d+\bar{d}} + \lambda xf_1^{s+\bar{s}} \right) A_p^{hh} + \left(5xf_1^{u+\bar{u}} + 5xf_1^{d+\bar{d}} + 2\lambda xf_1^{s+\bar{s}} \right) A_d^{hh} \right], \quad (12)$$

$$xh_1^{dv} = \frac{1}{15} \frac{1}{\tilde{a}_P^{hh}} \left[-3 \left(4xf_1^{u+\bar{u}} + xf_1^{d+\bar{d}} + \lambda xf_1^{s+\bar{s}} \right) A_p^{hh} + 4 \left(5xf_1^{u+\bar{u}} + 5xf_1^{d+\bar{d}} + 2\lambda xf_1^{s+\bar{s}} \right) A_d^{hh} \right] \quad (13)$$

B. Dihadron asymmetries in e^+e^- annihilation

Following the original treatment of Ref. [21] we will extract the analyzing power \tilde{a}_P^{hh} from the measurement of the production of dihadron pairs in electron-positron annihilation: $e^+e^- \rightarrow (h_1 h_2) (h'_1 h'_2) X$, where the particles in brackets belong to two back-to-back jets. The relevant quantity is the angular correlation of the production planes, expressed by the so-called Artru-Collins asymmetry [31, 32]. The kinematics of the process is described by doubling the variables previously introduced. All the variables related to the second jet (initiated by the antiquark) will be denoted by a bar.

If we call ϕ_R and $\bar{\phi}_R$ the azimuthal angles of the transverse relative momenta \mathbf{R}_T and $\bar{\mathbf{R}}_T$ of the two dihadrons, the Artru-Collins azimuthal asymmetry is the amplitude of the $\cos(\phi_R + \bar{\phi}_R)$ modulation and reads [21, 26, 29] (this quantity is called a_{12} in Ref. [26])

$$A_{e^+e^-}^{hh}(z_{hh}, M_{hh}^2, \bar{z}_{hh}, \bar{M}_{hh}^2, Q^2) = -\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \times \frac{\sum_{q,\bar{q}} e_q^2 \frac{|\mathbf{R}_T|}{M_{hh}} H_{1q}^{\lessdot}(z_{hh}, M_{hh}^2, Q^2) \frac{|\bar{\mathbf{R}}_T|}{\bar{M}_{hh}} H_{1\bar{q}}^{\lessdot}(\bar{z}_{hh}, \bar{M}_{hh}^2, Q^2)}{\sum_{q,\bar{q}} e_q^2 D_{1q}^{hh}(z_{hh}, M_{hh}^2, Q^2) D_{1\bar{q}}^{hh}(\bar{z}_{hh}, \bar{M}_{hh}^2, Q^2)} \quad (14)$$

where θ_2 is the angle between the e^+ and the thrust axis.

Incorporating $|\mathbf{R}_T|/M_{hh}$ and $|\bar{\mathbf{R}}_T|/\bar{M}_{hh}$ into the interference dihadron fragmentation function and integrating over z_{hh}, \bar{z}_{hh} and M_{hh}^2, \bar{M}_{hh}^2 , the Artru-Collins asymmetry becomes

$$A_{e^+e^-}^{hh}(Q^2) = -\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \frac{\sum_{q,\bar{q}} e_q^2 \tilde{H}_{1q}^{\lessdot}(Q^2) \tilde{H}_{1\bar{q}}^{\lessdot}(Q^2)}{\sum_{q,\bar{q}} e_q^2 \tilde{D}_{1q}^{hh}(Q^2) \tilde{D}_{1\bar{q}}^{hh}(Q^2)} \quad (15)$$

We can simplify this expression by using Eqs. (5) and (6) and the relation $\tilde{D}_{1s}^{hh} = \lambda \tilde{D}_{1u}^{hh}$. As for the charm, the Belle experiment [26] finds that the c yield is about one half the uds , with a 10% uncertainty. Thus we set

$$e_c^2 \tilde{D}_{1c}^{hh} \tilde{D}_{1\bar{c}}^{hh} = \mu^2 \left(e_u^2 \tilde{D}_{1u}^{hh} \tilde{D}_{1\bar{u}}^{hh} + e_d^2 \tilde{D}_{1d}^{hh} \tilde{D}_{1\bar{d}}^{hh} + e_s^2 \tilde{D}_{1s}^{hh} \tilde{D}_{1\bar{s}}^{hh} \right), \quad (16)$$

with $\mu^2 = 0.5$.

Finally we get

$$\sum_{q,\bar{q}} e_q^2 \tilde{H}_{1q}^{\lessdot} \tilde{H}_{1\bar{q}}^{\lessdot} = -\frac{10}{9} \left(\tilde{H}_{1u}^{\lessdot} \right)^2, \quad (17)$$

$$\sum_{q,\bar{q}} e_q^2 \tilde{D}_{1q}^{hh} \tilde{D}_{1\bar{q}}^{hh} = \frac{2}{9} (1 + \mu^2) (5 + \lambda^2) \left(\tilde{D}_{1u}^{hh} \right)^2. \quad (18)$$

Using eqs. (17,18), the Artru-Collins asymmetry takes the form

$$A_{e^+e^-}^{hh}(Q^2) = -\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \frac{5}{(1 + \mu^2)(5 + \lambda^2)} \left[\frac{\tilde{H}_{1u}^{\lessdot}(Q^2)}{\tilde{D}_{1u}^{hh}(Q^2)} \right]^2. \quad (19)$$

Solving for the analyzing power, we obtain

$$|\tilde{a}_P^{hh}(Q^2)| = \left| \frac{\tilde{H}_{1u}^{\lessdot}(Q^2)}{\tilde{D}_{1u}^{hh}(Q^2)} \right| = \sqrt{-\frac{1}{5} (1 + \mu^2) (5 + \lambda^2) \frac{\langle 1 + \cos^2 \theta_2 \rangle}{\langle \sin^2 \theta_2 \rangle} A_{e^+e^-}^{hh}(Q^2)}, \quad (20)$$

where the overall sign, which is left undetermined by the e^+e^- data, will be chosen in such a way to obtain the expected final sign of the transversity distributions, that is a positive h_1^{uv} . With $\lambda = 0.5$, $\mu^2 = 0.5$ and the numerical values [26]

$$\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} = 0.7636, \quad A_{e^+e^-}^{hh} = -0.0196 \pm 0.0002 \pm 0.0022 \quad (21)$$

we find

$$|\tilde{a}_P^{hh}(Q_B^2)| = 0.201 \quad \text{at} \quad Q_B^2 \simeq 110 \text{ GeV}^2/c^2 \quad (22)$$

with a negligible statistical error and a relative systematic uncertainty of about 5%. We also explored another hypothesis for the strange and charm contribution to \tilde{D}_1^{hh} , that is $\tilde{D}_{1s}^{hh} = \tilde{D}_{1c}^{hh}$ and $\tilde{D}_{1s}^{hh} = 0.8 \tilde{D}_{1u}^{hh}$, which is also compatible with the Belle finding for the c yield. The final result for \tilde{a}_P^{hh} decreases by 5%.

	COMPASS p	COMPASS d	Belle
$\langle z_{hh} \rangle$	0.439–0.479	0.434–0.485	0.4313
$\langle M_{hh} \rangle$ (GeV/c ²)	0.654–0.723	0.633–0.722	0.6186

TABLE I: Mean values of z_{hh} and M_{hh} at COMPASS and Belle. For COMPASS the ranges refer to the different x bins.

$\langle x \rangle$	Q^2 (GeV ² /c ²)	$xh_1^{u_v}$	$xh_1^{d_v}$
0.006	1.23	-0.04 ± 0.04	0.08 ± 0.12
0.010	1.48	0.03 ± 0.02	-0.06 ± 0.07
0.016	1.74	0.07 ± 0.02	0.17 ± 0.05
0.025	2.09	0.02 ± 0.02	-0.06 ± 0.05
0.040	2.80	0.05 ± 0.02	0.00 ± 0.07
0.062	4.34	0.06 ± 0.03	-0.12 ± 0.09
0.100	6.85	0.09 ± 0.04	-0.38 ± 0.12
0.161	10.7	0.15 ± 0.05	-0.30 ± 0.19
0.280	22.0	0.25 ± 0.06	0.26 ± 0.23

TABLE II: Values of the valence transversity distributions. Note that the Q^2 values refer to the proton data. The deuteron data are taken at slightly larger Q^2 , and in the last bin it is $Q^2 = 33.2$ GeV²/c². Errors are statistical only.

C. Extraction of the transversity distributions

The value of the analyzing power obtained from e^+e^- data will be used for the extraction of the transversity distributions from COMPASS data without any correction, since the mean values of z_{hh} and M_{hh} in COMPASS are quite close to those of Belle, as shown in Table I. Concerning Q^2 , this is quite different from Belle to COMPASS. However, for the purposes of this paper we neglect the Q^2 evolution of the analyzing power, which has been shown to introduce only a few percent effect [21].

Using the dihadron asymmetries measured by COMPASS (the produced hadrons are assumed to be pions), the analyzing power (22) extracted from Belle data, and the unpolarized PDF's $f_1^{u+\bar{u}}$ and $f_1^{d+\bar{d}}$ from the CTEQ5D global fit [33], we obtain from Eqs. (9) and (10) the combinations $4xh_1^{u_v} - xh_1^{d_v}$ and $xh_1^{u_v} + xh_1^{d_v}$, which are shown in Fig. 1. The uncertainties shown are statistical only and no uncertainty on \tilde{a}_P^{hh} has been considered. We found that the results depend very little on the magnitude of the strange contribution, that is on the value of λ .

In Fig. 1 we also plotted the same combinations of the transversity distributions found by the Pavia group [22]. As one can see, the results of our approach are close to those obtained in Ref. [22]. The differences in the combination $4xh_1^{u_v} - xh_1^{d_v}$, which is determined by the proton data, are mainly due to the fact that in Ref. [22] only the 2007 COMPASS data were used, while the present analysis relies on a much wider data set, which includes the published 2007 and 2010 data.

From Eqs. (12) and (13) we get the transversity distributions for each flavor separately. These are displayed in Fig. 2 and their values are collected in Table II, with the corresponding Q^2 . It turns out that $h_1^{u_v}$ is rather well determined, much better than $h_1^{d_v}$, due to the larger uncertainties of the deuteron asymmetry, which dominates the $h_1^{d_v}$ extraction, as one can see from Eq. (13). The two distributions have opposite signs, with $h_1^{u_v}$ positive as a consequence of the sign choice in Eq. (22).

For comparison, in Fig. 2 we also show the results of one of the fits (the so-called “flexible scenario”) of Ref. [22] at $Q^2 = 2.4$ GeV²/c². Again, the agreement between our determination and that of the Pavia group is good, our points lying within the error bands of their fit. In the case of the d quark, the error band of the Pavia fit at large x shrinks into a line due to the constrain of the Soffer bound. The broad band for the u quark as compared with the statistical errors of the present result is very likely due to the larger uncertainties in the combinations $4xh_1^{u_v} - xh_1^{d_v}$ as already noted.

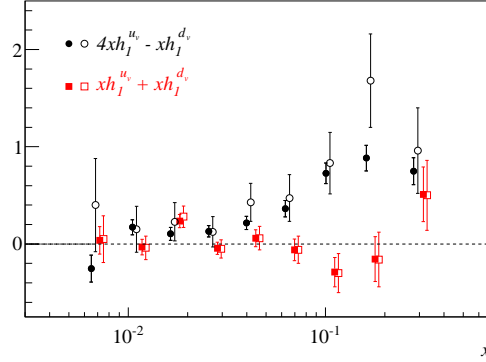


FIG. 1: The combinations $4xh_1^{uv} - xh_1^{dv}$ (black solid circles) and $xh_1^{uv} + xh_1^{dv}$ (red solid squares). For comparison we plot also the corresponding results of Ref. [22] at $Q^2 = 2.4 \text{ GeV}^2$ (open points). Squares and open circles are horizontally shifted to make them more visible.

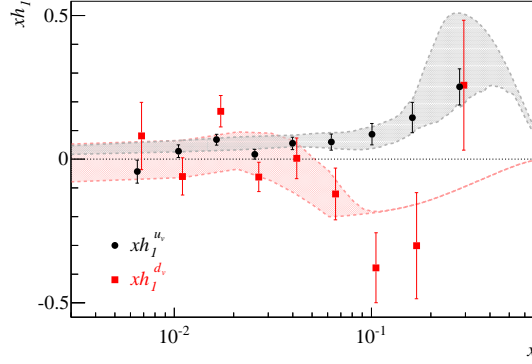


FIG. 2: The transversity distributions, xh_1^{uv} (black circles) and xh_1^{dv} (red squares) extracted from dihadron data. The shadowed areas are the results of the fits of the corresponding linear combinations shown in Fig. 1 from Ref. [22] at $Q^2 = 2.4 \text{ GeV}^2$, in the “flexible scenario”. For the meaning of the bands see Ref. [22].

III. COLLINS ASYMMETRIES

A. Collins asymmetries in SIDIS

The first evidence of the existence of transversity came from the experimental study of one-hadron inclusive lepton production from a transversely polarized proton target, $\ell p^\uparrow \rightarrow \ell' h X$, at HERMES [5]. The same signal was subsequently found at higher Q^2 by the COMPASS Collaboration [7].

We denote by P_h and M_h the momentum and the mass, respectively, of the produced hadron. Conventionally, all azimuthal angles are referred to the lepton scattering plane: ϕ_h is the azimuthal angle of the hadron h , ϕ_S is the azimuthal angle of the nucleon spin vector \mathbf{S}_\perp . The transverse momenta are defined as follows: \mathbf{k}_T is the transverse momentum of the quark inside the nucleon, \mathbf{p}_T is the transverse momentum of the hadron with respect to the fragmenting quark, $\mathbf{P}_{h\perp}$ is the measurable transverse momentum of the produced hadron with respect to the γ^*N axis.

The $\sin(\phi_h + \phi_S)$ term in the cross section couples the transverse-momentum dependent transversity distribution $h_1(x, k_T^2, Q^2)$ to the Collins function $H_1^\perp(z, p_T^2, Q^2)$, which describes the fragmentation of the

transversely polarized struck quark into a spinless hadron. The corresponding asymmetry is

$$A^h(x, z, Q^2) = \frac{\int d^2 \mathbf{P}_{h\perp} \mathcal{C} \left[\frac{\mathbf{P}_{h\perp} \cdot \mathbf{p}_T}{z M_h P_{h\perp}} h_1 H_1^\perp \right]}{\int d^2 \mathbf{P}_{h\perp} \mathcal{C} [f_1 D_1]}, \quad (23)$$

where the convolution \mathcal{C} is defined as

$$\begin{aligned} \mathcal{C} [w f D] &= \sum_a e_a^2 x \int d^2 \mathbf{k}_T \int d^2 \mathbf{p}_T \delta^2(z \mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{h\perp}) \\ &\times w(\mathbf{k}_T, \mathbf{p}_T) f^a(x, k_T^2, Q^2) D^a(z, p_T^2, Q^2). \end{aligned} \quad (24)$$

If we use, as it is commonly done, a Gaussian Ansatz for the transverse-momentum dependent distributions

$$f_1(x, k_T^2, Q^2) = f_1(x, Q^2) \frac{e^{-k_T^2 / \langle k_T^2 \rangle}}{\pi \langle k_T^2 \rangle}, \quad h_1(x, k_T^2, Q^2) = h_1(x, Q^2) \frac{e^{-k_T^2 / \langle k_T^2 \rangle}}{\pi \langle k_T^2 \rangle}, \quad (25)$$

and fragmentation functions

$$D_1^\perp(z, p_T^2, Q^2) = D_1(z, Q^2) \frac{e^{-p_T^2 / \langle p_T^2 \rangle}}{\pi \langle p_T^2 \rangle}, \quad H_1^\perp(z, p_T^2, Q^2) = H_1^\perp(z, Q^2) \frac{e^{-p_T^2 / \langle p_T^2 \rangle}}{\pi \langle p_T^2 \rangle}, \quad (26)$$

the Collins asymmetry (23) becomes [34]

$$A^h(x, z, Q^2) = G(z) \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q(x, Q^2) H_{1q}^{\perp(1/2)}(z, Q^2)}{\sum_{q, \bar{q}} e_q^2 x f_1^q(x, Q^2) D_{1q}(z, Q^2)} \quad (27)$$

where

$$G(z) = \frac{1}{\sqrt{1 + z^2 \langle k_T^2 \rangle / \langle p_T^2 \rangle}}. \quad (28)$$

The “half-moment” of H_1^\perp is defined as

$$H_1^{\perp(1/2)}(z, Q^2) \equiv \int d^2 \mathbf{p}_T \frac{p_T}{z M_h} H_1^\perp(z, p_T^2, Q^2), \quad (29)$$

and in the Gaussian model is proportional to $H_1^\perp(z, Q^2)$, as defined in Eq. (26):

$$H_1^{\perp(1/2)}(z, Q^2) = \frac{\sqrt{\pi \langle p_T^2 \rangle}}{2z M_h} H_1^\perp(z, Q^2). \quad (30)$$

In the following we will set $G(z) = 1$, assuming $z^2 \langle k_T^2 \rangle / \langle p_T^2 \rangle \ll 1$. This assumption is expected to be reasonable, especially at low z , where the statistics is higher.

Being interested in the extraction of the transversity distributions, we can integrate over z ,

$$\tilde{H}_1^{\perp(1/2)}(Q^2) = \int dz H_1^{\perp(1/2)}(z, Q^2), \quad \tilde{D}_1(Q^2) = \int dz D_1(z, Q^2), \quad (31)$$

and write the integrated asymmetry as

$$A^\pm(x, Q^2) = \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q(x, Q^2) \tilde{H}_{1q}^{\perp(1/2)\pm}(Q^2)}{\sum_{q, \bar{q}} e_q^2 x f_1^q(x, Q^2) \tilde{D}_{1q}^\pm(Q^2)}. \quad (32)$$

where the superscripts $+$ and $-$ denote the asymmetries and the fragmentation functions for π^+ and π^- production, respectively.

One usually distinguishes favored and unfavored fragmentation functions, defined as

$$D_{1,\text{fav}} = D_{1u}^+ = D_{1d}^- = D_{1\bar{u}}^- = D_{1\bar{d}}^+ \quad (33)$$

$$D_{1,\text{unf}} = D_{1u}^- = D_{1d}^+ = D_{1\bar{u}}^+ = D_{1\bar{d}}^- = D_{1s}^\pm = D_{1\bar{s}}^\pm. \quad (34)$$

The corresponding relations for H_1^\perp are

$$H_{1,\text{fav}}^\perp = H_{1u}^{\perp+} = H_{1d}^{\perp-} = H_{1\bar{u}}^{\perp-} = H_{1\bar{d}}^{\perp+} \quad (35)$$

$$H_{1,\text{unf}}^\perp = H_{1u}^{\perp-} = H_{1d}^{\perp+} = H_{1\bar{u}}^{\perp+} = H_{1\bar{d}}^{\perp-}. \quad (36)$$

We assume $H_{1s}^\perp = H_{1\bar{s}}^\perp = 0$, as suggested by the string model [35], and we ignore the c components of the distribution functions, which are negligible at the x, Q^2 values of interest here. The denominators of the asymmetries $\sum_{q,\bar{q}} e_q^2 x f_1^q \tilde{D}_{1q}$, for a proton and a deuteron target (p, d) and for charged pions, multiplied by 9, can be rewritten as

$$p, \pi^+ : \quad x [4(f_1^u + \tilde{\beta} f_1^{\bar{u}}) + (\tilde{\beta} f_1^d + f_1^{\bar{d}}) + \tilde{\beta}(f_1^s + f_1^{\bar{s}})] \tilde{D}_{1,\text{fav}} \equiv x f_p^+ \tilde{D}_{1,\text{fav}}, \quad (37)$$

$$d, \pi^+ : \quad x [(4 + \tilde{\beta})(f_1^u + f_1^d) + (1 + 4\tilde{\beta})(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2\tilde{\beta}(f_1^s + f_1^{\bar{s}})] \tilde{D}_{1,\text{fav}} \equiv x f_d^+ \tilde{D}_{1,\text{fav}}, \quad (38)$$

$$p, \pi^- : \quad x [4(\tilde{\beta} f_1^u + f_1^{\bar{u}}) + (f_1^d + \tilde{\beta} f_1^{\bar{d}}) + \tilde{\beta}(f_1^s + f_1^{\bar{s}})] \tilde{D}_{1,\text{fav}} \equiv x f_p^- \tilde{D}_{1,\text{fav}}, \quad (39)$$

$$d, \pi^- : \quad x [(1 + 4\tilde{\beta})(f_1^u + f_1^d) + (4 + \tilde{\beta})(f_1^{\bar{u}} + f_1^{\bar{d}}) + 2\tilde{\beta}(f_1^s + f_1^{\bar{s}})] \tilde{D}_{1,\text{fav}} \equiv x f_d^- \tilde{D}_{1,\text{fav}}, \quad (40)$$

where

$$\tilde{\beta}(Q^2) = \frac{\tilde{D}_{1,\text{unf}}(Q^2)}{\tilde{D}_{1,\text{fav}}(Q^2)} \quad (41)$$

can be taken from standard parametrizations of fragmentation functions.

Similar expressions (with no strange terms) are obtained for the numerator of Eq. (32), $\sum_{q,\bar{q}} e_q^2 x h_1^q \tilde{H}_{1q}^{\perp(1/2)}$, with the replacements $\tilde{D}_1 \rightarrow \tilde{H}_1^\perp$, $f_1 \rightarrow h_1$, and $\tilde{\beta} \rightarrow \tilde{\alpha}$, where

$$\tilde{\alpha}(Q^2) = \frac{\tilde{H}_{1,\text{unf}}^{\perp(1/2)}(Q^2)}{\tilde{H}_{1,\text{fav}}^{\perp(1/2)}(Q^2)} \quad (42)$$

is unknown and will be determined later by means of some assumptions.

Introducing the analyzing power

$$\tilde{a}_P^h(Q^2) = \frac{\tilde{H}_{1,\text{fav}}^{\perp(1/2)}(Q^2)}{\tilde{D}_{1,\text{fav}}(Q^2)}, \quad (43)$$

we find for the proton target

$$A_p^+ = \tilde{a}_P^h \frac{4(h_1^u + \tilde{\alpha} h_1^{\bar{u}}) + (\tilde{\alpha} h_1^d + h_1^{\bar{d}})}{f_p^+}, \quad (44)$$

$$A_p^- = \tilde{a}_P^h \frac{4(\tilde{\alpha} h_1^u + h_1^{\bar{u}}) + (h_1^d + \tilde{\alpha} h_1^{\bar{d}})}{f_p^-}, \quad (45)$$

and for the deuteron target

$$A_d^+ = \tilde{a}_P^h \frac{(4 + \tilde{\alpha})(h_1^u + h_1^d) + (1 + 4\tilde{\alpha})(h_1^{\bar{u}} + h_1^{\bar{d}})}{f_d^+}, \quad (46)$$

$$A_d^- = \tilde{a}_P^h \frac{(1 + 4\tilde{\alpha})(h_1^u + h_1^d) + (4 + \tilde{\alpha})(h_1^{\bar{u}} + h_1^{\bar{d}})}{f_d^-}. \quad (47)$$

The combinations

$$f_p^+ A_p^+ - f_p^- A_p^- = \tilde{a}_P^h (1 - \tilde{\alpha}) (4h_1^{u_v} - h_1^{d_v}) \quad (48)$$

$$f_d^+ A_d^+ - f_d^- A_d^- = \tilde{a}_P^h 3(1 - \tilde{\alpha}) (h_1^{u_v} + h_1^{d_v}) \quad (49)$$

select the valence transversity distributions. From eqs. (48, 49), we get the valence distributions for u and d quarks separately:

$$xh_1^{u_v} = \frac{1}{5} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha})} \left[(xf_p^+ A_p^+ - xf_p^- A_p^-) + \frac{1}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) \right], \quad (50)$$

$$xh_1^{d_v} = \frac{1}{5} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha})} \left[\frac{4}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) - (xf_p^+ A_p^+ - xf_p^- A_p^-) \right]. \quad (51)$$

Notice that the other two linearly independent combinations of the proton and deuteron asymmetries provide the valence + sea transversity distributions $h_1^{u+\bar{u}} = h_1^u + h_1^{\bar{u}}$ and $h_1^{d+\bar{d}} = h_1^d + h_1^{\bar{d}}$:

$$xh_1^{u+\bar{u}} = \frac{1}{3} \frac{1}{\tilde{a}_P^h (1 + \tilde{\alpha})} \left[(xf_p^+ A_p^+ + xf_p^- A_p^-) - \frac{1}{5} (xf_d^+ A_d^+ + xf_d^- A_d^-) \right], \quad (52)$$

$$xh_1^{d+\bar{d}} = \frac{1}{3} \frac{1}{\tilde{a}_P^h (1 + \tilde{\alpha})} \left[\frac{4}{5} (xf_d^+ A_d^+ + xf_d^- A_d^-) - (xf_p^+ A_p^+ + xf_p^- A_p^-) \right]. \quad (53)$$

By further combining Eqs. (50) and (51), and Eqs. (52) and (53), one can isolate the sea distributions:

$$xh_1^{\bar{u}} = \frac{1}{15} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha}^2)} \left[(1 - 4\tilde{\alpha}) xf_p^+ A_p^+ + (4 - \tilde{\alpha}) xf_p^- A_p^- - xf_d^+ A_d^+ + \tilde{\alpha} xf_d^- A_d^- \right], \quad (54)$$

$$xh_1^{\bar{d}} = \frac{1}{15} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha}^2)} \left[(4\tilde{\alpha} - 1) xf_p^+ A_p^+ - (4 - \tilde{\alpha}) xf_p^- A_p^- - 4\tilde{\alpha} xf_d^+ A_d^+ + 4 xf_d^- A_d^- \right]. \quad (55)$$

The overall sea transversity, $h_1^{\bar{u}} + h_1^{\bar{d}}$, is determined by the deuteron asymmetries only:

$$xh_1^{\bar{u}} + xh_1^{\bar{d}} = \frac{1}{15} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha}^2)} \left[(4 + \tilde{\alpha}) xf_d^- A_d^- - (4\tilde{\alpha} + 1) xf_d^+ A_d^+ \right]. \quad (56)$$

B. Collins asymmetries in e^+e^- annihilation

The analyzing power \tilde{a}_P^h is obtained from inclusive two-hadron production in electron-positron annihilation, $e^+e^- \rightarrow h_1 h_2 X$, with the two hadrons in different hemispheres. In this process the Collins effect is observed in the combination of the fragmenting processes of a quark and an antiquark, resulting in the product of two Collins functions with an overall modulation of the type $\cos(\phi_1 + \phi_2)$, where ϕ_1 and ϕ_2 are the azimuthal angles of the final hadrons around the quark-antiquark axis (approximated by the thrust axis), with respect to the $e^+e^- \rightarrow q\bar{q}$ scattering plane.

The resulting $\cos(\phi_1 + \phi_2)$ asymmetry is given by [24, 25, 36]

$$A_{e^+e^-}^h(z_1, z_2, Q^2) = \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} \frac{\sum_q e_q^2 H_{1q}^{\perp(1/2)}(z_1, Q^2) H_{1\bar{q}}^{\perp(1/2)}(z_2, Q^2)}{\sum_q e_q^2 D_{1q}(z_1, Q^2) D_{1\bar{q}}(z_2, Q^2)}, \quad (57)$$

having denoted by $z_1(z_2)$ the fraction of the light-cone momentum of the quark (antiquark) carried by the produced hadron.

The quantity measured in practice is the difference of the asymmetries for unlike-sign (U) and like-sign (L) pion pairs, i.e.

$$A_{e^+e^-}^{\text{UL}}(z_1, z_2, Q^2) = \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} [A_U(z_1, z_2, Q^2) - A_L(z_1, z_2, Q^2)], \quad (58)$$

where A_U and A_L , in terms of the favored and unfavored fragmentation functions defined as in Eqs. (33) and (34), are explicitly given by [18]

$$A_U(z_1, z_2, Q^2) = \frac{5 H_{1,\text{fav}}^{\perp(1/2)}(z_1, Q^2) H_{1,\text{fav}}^{\perp(1/2)}(z_2, Q^2) + 5 H_{1,\text{unf}}^{\perp(1/2)}(z_1, Q^2) H_{1,\text{unf}}^{\perp(1/2)}(z_2, Q^2)}{5 D_{1,\text{fav}}(z_1, Q^2) D_{1,\text{fav}}(z_2, Q^2) + 7 D_{1,\text{unf}}(z_1, Q^2) D_{1,\text{unf}}(z_2, Q^2)} \quad (59)$$

$$A_L(z_1, z_2, Q^2) = \frac{5 \left[H_{1,\text{fav}}^{\perp(1/2)}(z_1, Q^2) H_{1,\text{unf}}^{\perp(1/2)}(z_2, Q^2) + (z_1 \leftrightarrow z_2) \right]}{5 [D_{1,\text{fav}}(z_1, Q^2) D_{1,\text{unf}}(z_2, Q^2) + (z_1 \leftrightarrow z_2)] + 2 D_{1,\text{unf}}(z_1, Q^2) D_{1,\text{unf}}(z_2, Q^2)} \quad (60)$$

Note that the Belle data are subtracted for charm, hence the c components of the fragmentation functions have been ignored.

Let us now introduce the unfavored-to-favored ratios of fragmentation functions

$$\alpha(z_i, Q^2) = \frac{H_{1,\text{unf}}^{\perp(1/2)}(z_i, Q^2)}{H_{1,\text{fav}}^{\perp(1/2)}(z_i, Q^2)}, \quad \beta(z_i, Q^2) = \frac{D_{1,\text{unf}}(z_i, Q^2)}{D_{1,\text{fav}}(z_i, Q^2)}, \quad z_i = z_1, z_2. \quad (61)$$

The corresponding integrated quantities have already been defined in Eqs. (42) and (41).

If we set $z_1 = z_2 \equiv z$, that is we use only the diagonal measurements, the UL asymmetry can be put in the form

$$A_{e^+e^-}^{\text{UL}}(z, Q^2) = \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} \left[\frac{H_{1,\text{fav}}^{\perp(1/2)}(z, Q^2)}{D_{1,\text{fav}}(z, Q^2)} \right]^2 B(z, Q^2) \quad (62)$$

with

$$B = \frac{5 + 5\alpha^2}{5 + 7\beta^2} - \frac{5\alpha}{5\beta + \beta^2}. \quad (63)$$

Solving for the analyzing power, we get

$$|a_P^h(z, Q^2)| \equiv \left| \frac{H_{1,\text{fav}}^{\perp(1/2)}(z, Q^2)}{D_{1,\text{fav}}(z, Q^2)} \right| = \sqrt{A_{e^+e^-}^{\text{UL}}(z, Q^2) \frac{\langle 1 + \cos^2 \theta \rangle}{\langle \sin^2 \theta \rangle} \frac{1}{B(z, Q^2)}}. \quad (64)$$

While $\beta(z, Q^2)$ and $\tilde{\beta}(Q^2)$ can be obtained from standard parametrizations of unpolarized fragmentation functions, the functions $\alpha(z, Q^2)$ and $\tilde{\alpha}(Q^2)$ are not known. We consider two different scenarios for the relation between the unfavored and the favored Collins function.

- **Scenario 1.** In this scenario we assume that the unfavored Collins function is equal and opposite to the favored one,

$$H_{1,\text{fav}}^{\perp(1/2)}(z, Q^2) = -H_{1,\text{unf}}^{\perp(1/2)}(z, Q^2), \quad (65)$$

that is we set $\alpha(z, Q^2) = -1$. This assumption is suggested by the fact that the asymmetries for positive and negative pions are found to have approximately the same size but an opposite sign. The Schäfer–Teryaev sum rule [37] also points to the same conclusion if interpreted as a relation valid for each hadronic species separately [38].

For the ratio of the integrated unfavored to favored Collins function, Eq. (42), one also gets $\tilde{\alpha}(Q^2) = -1$. Notice that, in this case, all proton and deuteron asymmetries (44-47) depend on the valence transversity only, and the sea transversity is undetermined.

- **Scenario 2.** In this scenario we assume $\alpha(z, Q^2) = -\beta(z, Q^2)$, that is

$$\frac{H_{1,\text{unf}}^{\perp(1/2)}(z, Q^2)}{H_{1,\text{fav}}^{\perp(1/2)}(z, Q^2)} = -\frac{D_{1,\text{unf}}(z, Q^2)}{D_{1,\text{fav}}(z, Q^2)}, \quad (66)$$

or equivalently

$$\frac{H_{1,\text{fav}}^{\perp(1/2)}(z, Q^2)}{D_{1,\text{fav}}(z, Q^2)} = -\frac{H_{1,\text{unf}}^{\perp(1/2)}(z, Q^2)}{D_{1,\text{unf}}(z, Q^2)}, \quad (67)$$

z	$A_{e^+e^-}^{\text{UL}}$	$a_P^h(1)$	$a_P^h(2)$
0.244	0.010 ± 0.011	0.071 ± 0.040	0.090 ± 0.050
0.377	0.046 ± 0.005	0.137 ± 0.007	0.189 ± 0.010
0.577	0.113 ± 0.006	0.178 ± 0.005	0.290 ± 0.008
0.781	0.206 ± 0.024	0.170 ± 0.010	0.387 ± 0.023

TABLE III: The e^+e^- asymmetry measured by Belle [25] (with statistical and systematic errors added in quadrature) and the resulting analyzing powers in scenarios 1 and 2.

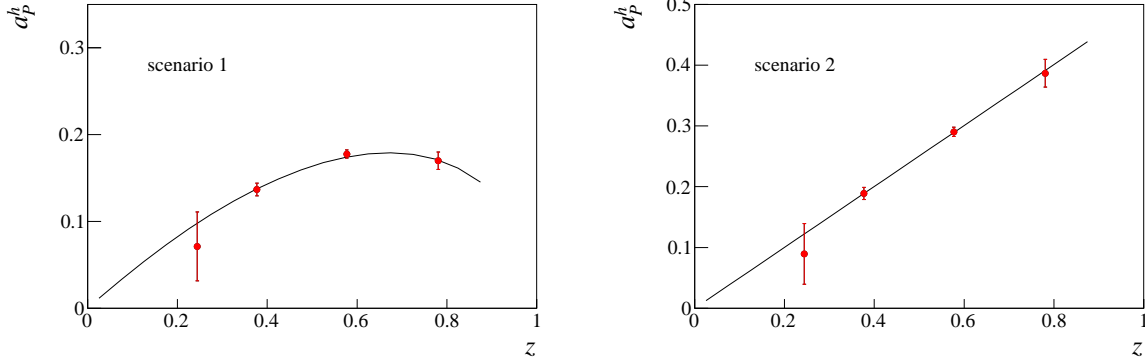


FIG. 3: The analyzing power a_P^h at $Q_B^2 = 110 \text{ GeV}^2/c^2$ as a function of z for scenario 1 (left) and 2 (right). The curves are described in the text.

These two scenarios, which lead to quite different expressions for $B(z, Q^2)$, can be regarded as a measure of the uncertainty related to the lack of knowledge of the different components of the Collins function. We will see, however, that the final results are quite insensitive to this choice.

We get $D_{1,\text{unf}}$ and $D_{1,\text{fav}}$ from the DSS parametrization of fragmentation functions [39]. In the DSS fit, actually, D_{1u}^+ is not taken to be equal to $D_{1\bar{d}}^+$, but their difference is rather small. Thus, we identify $D_{1,\text{fav}}$ with $(D_{1u}^+ + D_{1\bar{d}}^+)/2$ as given by DSS.

The experimental values of $A_{e^+e^-}^{\text{UL}}$ at $z_1 = z_2 = z$ [25] are given in Table III together with the resulting analyzing powers in the two scenarios, which are also shown in Fig. 3. The value $\langle \sin^2 \theta \rangle / \langle 1 + \cos^2 \theta \rangle = 0.70$ has been used [25].

Since we need to integrate over z in order to obtain $\tilde{a}_P^h(Q^2)$, we must interpolate in z the values of $a_P^h(z, Q^2)$ listed in Table III. We use the following very simple fitting functions:

- Scenario 1:

$$a_P^h(z, Q_B^2) = Nz(1-z)^\gamma, \quad Q_B^2 = 110 \text{ GeV}^2/c^2, \quad (68)$$

with $N = 0.46 \pm 0.03$ and $\gamma = 0.49 \pm 0.07$.

- Scenario 2:

$$a_P^h(z, Q_B^2) = N'z, \quad Q_B^2 = 110 \text{ GeV}^2/c^2. \quad (69)$$

with $N' = 0.501 \pm 0.011$.

These curves are shown in Fig. 3.

C. Extraction of the transversity distributions

In order to evaluate the analyzing power for the Collins asymmetry at the values of Q^2 of the COMPASS x -bins, we need to evolve the fragmentation functions from the Belle value $Q_B^2 = 110 \text{ GeV}^2/c^2$ to the

Q^2 values of COMPASS data. The evolution of $H_1^{\perp(1/2)}(z, Q^2)$ has been worked out in Refs. [40–42], but involves unknown twist-3 fragmentation functions. For our purpose we resort to some simplifying assumptions. As a first hypothesis, consistently with what we have done in the dihadron case, we assume the analyzing power $a_P^h(z, Q^2)$ to be constant in Q^2 (another hypothesis for the evolution of the Collins function will be discussed later). Thus we have

$$\tilde{a}_P^h(Q^2) = \tilde{a}_P^h(Q_B^2) = \frac{\int dz a_P^h(z, Q_B^2) D_{1,\text{fav}}(z, Q_B^2)}{\int dz D_{1,\text{fav}}(z, Q_B^2)}. \quad (70)$$

The values of \tilde{a}_P^h for scenarios 1 and 2 are

$$\text{Scenario 1 : } \tilde{a}_P^h = 0.122, \quad \text{Scenario 2 : } \tilde{a}_P^h = 0.173. \quad (71)$$

It is now possible to extract the transversity distributions from the Collins asymmetries $A_{p/d}^\pm$. The last ingredient we need is the ratio $\alpha(Q^2)$ of the integrated unfavored to favored Collins functions, Eq. (42). Whereas for scenario 1 we have $\tilde{\alpha}(Q^2) = -1$, for scenario 2, using $\alpha(z, Q^2) = -\beta(z, Q^2)$ and the linear behavior of Eq. (69), we find

$$\tilde{\alpha}(Q^2) = -\frac{\int dz z D_{1,\text{unf}}(z, Q^2)}{\int dz z D_{1,\text{fav}}(z, Q^2)}. \quad (72)$$

In the COMPASS Q^2 range, $\tilde{\alpha}$ ranges from -0.43 at the highest x value to -0.34 at the lowest x value.

Using the CTEQ5D unpolarized distribution functions [33] and the DSS unpolarized fragmentation functions, and inserting the values (71) and the asymmetries measured by COMPASS into Eqs. (50) and (51), we finally find the valence transversity distributions plotted in Fig. 4 (left). As one can see the distributions for the two Scenarios are very close to each other. This is due to the fact that the different assumptions for the relation between the favored and the unfavored Collins functions, leading to a different $\tilde{\alpha}$, are compensated by the difference in the analyzing powers \tilde{a}_P^h extracted from the e^+e^- data in the two scenarios, so that the product $\tilde{a}_P^h(1 - \tilde{\alpha})$, which would discriminate between the two scenarios, is actually almost the same. Very much as in the dihadron case the valence u quark transversity distribution is definitively positive and well determined while the d quark has about the same size but opposite sign and has considerably larger uncertainties.

To check the robustness of our results also against a different hypothesis on the evolution of the fragmentation functions, we now assume that $H_{1,\text{fav}}^\perp$ evolves very little compared to $D_{1,\text{fav}}$, which implies, instead of Eq. (70),

$$\tilde{a}_P^h(Q^2) = \frac{\int dz a_P^h(z, Q_B^2) D_{1,\text{fav}}(z, Q_B^2)}{\int dz D_{1,\text{fav}}(z, Q^2)}. \quad (73)$$

We find that the resulting transversity distributions are close to those obtained under the assumption of Eq. (70), as can be seen in Fig. 4 (right). Their difference is of about 10% at large x .

In Fig. 5 we compare the results of the present paper with the transversity distributions extracted by the Torino group [20], at $Q^2 = 10 \text{ GeV}^2/c^2$. The effect of the evolution is however small, as shown by the solid and dashed lines, which refer to $Q^2 = 10 \text{ GeV}^2/c^2$ and $Q^2 = 2 \text{ GeV}^2/c^2$, respectively. Our point-by-point determination turns out to be in good agreement with the fit of Ref. [20] which includes also the HERMES proton data and all the asymmetries measured as functions of z and p_T . Recently, an extraction of the transversity using an approximate transverse-momentum dependent evolution at the next-to-leading logarithmic order has been performed [43], with results very similar to those found here.

In Scenario 2, where $\tilde{\alpha} \neq -1$, we can use Eqs. (54-56) to determine the sea transversity distributions. This is not possible in Scenario 1, however we regard the fact that the two scenarios lead to results which are essentially identical for the valence transversity as a justification to our procedure. The resulting values for the sea transversity distributions are shown in Fig. 6 (left) individually for \bar{u} and \bar{d} , while the combined distribution $xh_1^{\bar{u}} + xh_1^{\bar{d}}$, obtained from Eq. (56), is shown in Fig. 6 (right). Both the \bar{u} and the \bar{d} values are compatible with zero, but, as it is apparent from the results, the \bar{u} transversity values have an accuracy a factor of 3 better than the \bar{d} values.

The difference in sensitivity between the u and the d quark, which affects both the valence and the sea distributions, is due to the fact that the COMPASS deuteron data have larger errors than the proton

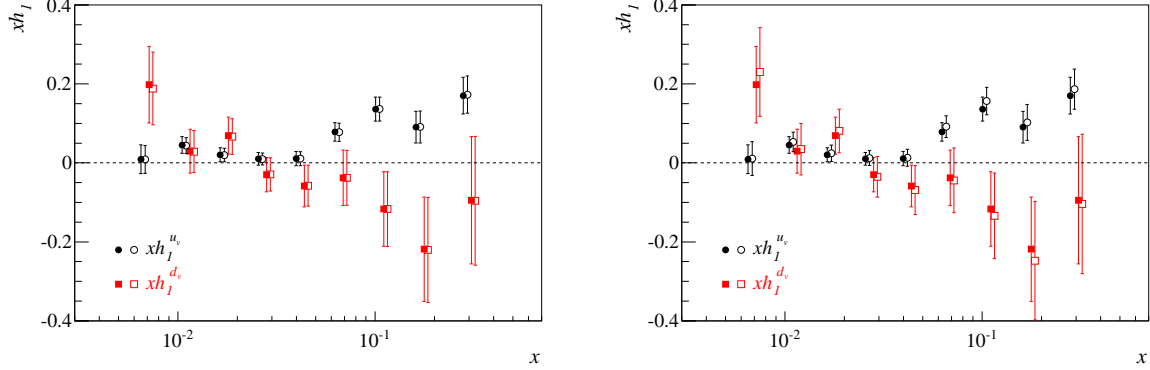


FIG. 4: Left: Valence transversity distributions for Scenario 1 (open points) and Scenario 2 (solid points). Right: Valence transversity distributions for Scenario 2 and the default, Eq. (70) (solid points), and alternative, Eq. (73) (open points), Q^2 evolution hypothesis. In both plots, black circles represent $xh_1^{u_v}$, red squares represent $xh_1^{d_v}$.

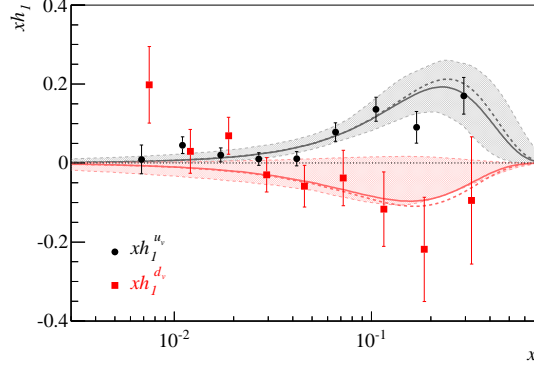


FIG. 5: Valence transversity distributions, $xh_1^{u_v}$ (black circles) and $xh_1^{d_v}$ (red squares) for Scenario 2, compared to the corresponding distributions extracted in Ref. [20] at $Q^2 = 10 \text{ GeV}^2/c^2$ (curves and error bands). The error bands refer to $Q^2 = 10 \text{ GeV}^2/c^2$. The solid line and the dashed line show the central values at $Q^2 = 10 \text{ GeV}^2/c^2$ and $Q^2 = 2 \text{ GeV}^2/c^2$, respectively. For the meaning of the bands see Ref. [20].

data. In the particular case of the sea distributions, from Eq. (56) it is clear that $xh_1^{\bar{u}} + xh_1^{\bar{d}}$ is determined directly from the Collins asymmetry of the deuteron data, which have rather large error bars. On the contrary, the \bar{u} distribution alone is well determined because in the right hand side of Eq. (54) the proton data have a considerably larger weight than the deuteron data.

All the values of the transversity distributions in the different x bins and the corresponding Q^2 are given in Table IV. The results from the extraction of transversity from the Collins asymmetries are the ones from Scenario 2, but as we have shown the two scenarios we have used led to essentially identical results.

IV. DISCUSSION OF THE RESULTS AND CONCLUDING REMARKS

We now compare the results for the transversity extracted from the dihadron asymmetries and from the Collins asymmetries in single-hadron leptonproduction. The valence transversity distributions obtained from these two types of processes are shown in Fig. 7. The agreement is quite impressive.

The error bars are computed from the statistical errors of the measured asymmetries as quoted by the

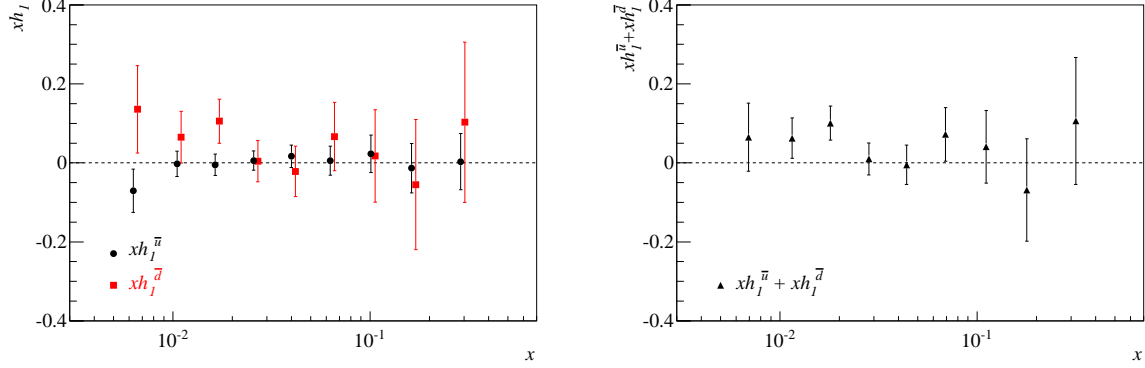


FIG. 6: The sea transversity distributions $xh_1^{\bar{u}}$ and $xh_1^{\bar{d}}$ (left) and $xh_1^{\bar{u}} + xh_1^{\bar{d}}$ (right) in Scenario 2.

$\langle x \rangle$	Q^2 (GeV $^2/c^2$)	$xh_1^{u_v}$	$xh_1^{d_v}$	$xh_1^{\bar{u}}$	$xh_1^{\bar{d}}$
0.006	1.27	0.01 ± 0.04	0.23 ± 0.11	-0.07 ± 0.05	0.14 ± 0.11
0.010	1.55	0.05 ± 0.03	0.03 ± 0.06	0.00 ± 0.03	0.07 ± 0.07
0.016	1.83	0.02 ± 0.02	0.08 ± 0.06	-0.01 ± 0.03	0.11 ± 0.06
0.025	2.17	0.01 ± 0.02	-0.03 ± 0.05	0.01 ± 0.02	0.00 ± 0.05
0.040	2.83	0.01 ± 0.02	-0.07 ± 0.06	0.02 ± 0.03	-0.02 ± 0.06
0.063	4.34	0.09 ± 0.03	-0.04 ± 0.08	0.00 ± 0.04	0.07 ± 0.09
0.101	6.76	0.16 ± 0.04	-0.13 ± 0.11	0.02 ± 0.05	0.02 ± 0.12
0.163	10.5	0.10 ± 0.04	-0.25 ± 0.15	-0.01 ± 0.06	-0.06 ± 0.17
0.288	22.6	0.19 ± 0.05	-0.10 ± 0.18	0.00 ± 0.07	0.10 ± 0.20

TABLE IV: Values of the valence and sea transversity distributions from the Collins asymmetries for Scenario 2. Note that the Q^2 values refer to the proton data. The deuteron data are taken at slightly larger Q^2 and in the last bin it is $Q^2 = 25.9$ GeV $^2/c^2$. Errors are statistical only.

experimental Collaborations, and no attempt has been made to try to assign a systematic error to the results. For the Collins extraction, the fact that different scenarios for the $H_{1,\text{fav}}^{\perp(1/2)}/H_{1,\text{unf}}^{\perp(1/2)}$ ratio and for the evolution lead to results which differ only by few percent is an indication that in our approach the

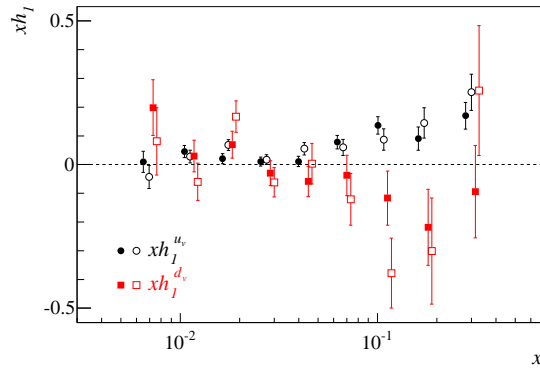


FIG. 7: The valence transversity distributions from dihadron (open points) and Collins asymmetries (solid points). Black circles represent $xh_1^{u_v}$, red squares represent $xh_1^{d_v}$. The transversity extracted from single-hadron leptonproduction refers to Scenario 2.

systematic uncertainties related to the phenomenological analysis are much smaller than the statistical errors. The transversity values obtained from the dihadron asymmetries and from the Collins asymmetries are very well compatible within themselves, and clearly support the fact that the same distributions are measured in the two processes.

Another relevant observation is that there is no indication for a bias due to the Gaussian ansatz and to the assumption $G = 1$ used to extract the transversity distributions from the Collins asymmetry.

It is also clear that the u quark transversity is determined with a much better accuracy than the d quark transversity, due to the fact that the asymmetry measurements on the proton are considerably more accurate than the corresponding ones on the deuteron, in particular in the valence region (the COMPASS Collaboration has taken data about 7 times less on deuterons than on protons). This unbalance can only increase if in a global analysis also the HERMES data are included, which were taken only on protons. Still, within the accuracy of the data, the d quark valence transversity distribution is definitively different from zero, and of about the same size as the corresponding u quark distribution, but with opposite sign.

Another interesting result from our work is the fact that our procedure has allowed also the extraction of the transversity distributions of the \bar{u} and the \bar{d} quarks. They are both found to be compatible with zero, but it is interesting to underline that the accuracy of this result in the case of \bar{u} is quite good, comparable to that of the u valence distribution.

To conclude, in a simple and direct model-independent way we have extracted the u and d quark transversity distributions, both valence and sea, from the COMPASS and the Belle data. The method seems robust, and probably can be extended to extract other distribution functions, in particular the Sivers and the Boer-Mulders functions.

To improve on the knowledge of transversity clearly more data are needed, in particular on the deuteron. The long-term solution to this quest is the planned future Electron Ion Collider (EIC), but in the near future measurements at JLAB12 and the proposed new COMPASS run on a deuteron target [44] will be highly beneficial.

Acknowledgments

One of us (V.B.) acknowledges the kind hospitality of the Dipartimento di Fisica of the University of Trieste, where this work was done. We acknowledge partial support from the Università degli Studi di Trieste in the framework of “Finanziamento di Ateneo per progetti di ricerca - FRA2012”. We are grateful to Stefano Melis and to Alessandro Bacchetta for providing us the curves of their fits.

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